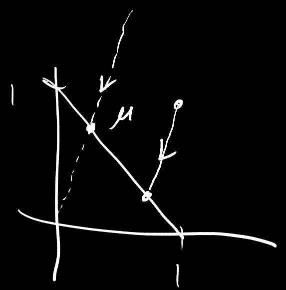

$$1^T\mu = 1$$

$$\Sigma = \begin{pmatrix} \sigma_1 & & 0 \\ & \ddots & \\ 0 & & \sigma_n \end{pmatrix}$$

$$P = I - \mu 1^T$$

$$P^2 = P$$

$$P\mu = 0$$

$$X = \mu + PZ \qquad Z \sim N(0, \Sigma)$$

$$E[X] = \mu \qquad cov(X) = cov(PZ).$$

$$= P \Sigma P^T$$

$$= \Sigma - \mu 1^T \Sigma - \Sigma 1 \mu^T + \mu 1^T \Sigma 1 \mu^T$$

$$1^T \Sigma = (\sigma_1 \cdots \sigma_n) \qquad \Sigma 1 = \begin{pmatrix} \sigma_1 \\ \vdots \\ \sigma_n \end{pmatrix}$$

$$1^T \Sigma 1 = \sigma_1 + \cdots + \sigma_n$$

$$c_i = diag(cov(X))_i = \sigma_i - 2\mu_i \sigma_i + (\sigma_1 + \cdots + \sigma_n)\mu_i^2$$

$$c = Q\sigma$$

$$Q = I - 2\, diag(\mu) + diag(\mu)^2 1 1^T$$

$$c = \sigma - 2\, diag(\mu)\sigma + diag(\mu)^2 1 1^T \sigma$$

$$D = diag(\mu)$$

$$c = \sigma - 2D\sigma + D^2 1 1^T \sigma$$

$$1^T \mu = 0 \qquad D = diag(\mu)$$

$$D1 = \mu \qquad 1^T D 1 = 1$$

$$1^T D = \mu^T$$

$$1^T c = 1^T \sigma - 2\, 1^T D \sigma + 1^T D^2 1 1^T \sigma$$

$$1^T c = 1^T \sigma - 2\mu^T \sigma + \mu^T \underset{\mu}{\cancel{D1}}\, 1^T \sigma$$

$$1^T c = 1^T \sigma - 2\mu^T \sigma + \mu^T \mu\, 1^T \sigma$$

$$= (1 + \mu^T \mu) 1^T \sigma - 2\mu^T \sigma$$

$$1^T \sigma = \frac{1^T c + 2\mu^T \sigma}{1 + \mu^T \mu}$$

$$c = (I - 2D)\sigma + D^2 1 \left(\frac{1^T c + 2\mu^T \sigma}{1 + \mu^T \mu} \right)$$

$$c = (I - 2D)\sigma + D^2 1 \left(\frac{1^T c + 2\mu^T \sigma}{1 + \mu^T \mu} \right)$$

$$D^2 1 \mu^T \sigma = D \mu \mu^T \sigma$$

$$Q = I - 2D + D^2 1 1^T$$

$$= \begin{pmatrix} 1 - 2\mu_1 + \mu_1^2 & \mu_1^2 & \mu_1^2 & \cdots \\ \mu_2^2 & 1 - 2\mu_2 + \mu_2^2 & \mu_2^2 & \cdots \\ \mu_3^2 & \mu_3^2 & 1 - 2\mu_3 + \mu_3^2 & \cdots \end{pmatrix}$$

$$c = Q\sigma$$

$$\begin{pmatrix} c_1 \\ | \\ c_n \end{pmatrix} = \begin{pmatrix} (1 - 2\mu_1)\sigma_1 \\ | \\ (1 - 2\mu_n)\sigma_n \end{pmatrix} + \begin{pmatrix} \mu_1^2 (\sigma_1 + \cdots + \sigma_n) \\ | \\ \mu_n^2 (\sigma_1 + \cdots + \sigma_n) \end{pmatrix}$$

$$\sigma_1 = \frac{1}{1 - 2\mu_1} c_1 + \frac{\mu_1^2}{1 - 2\mu_1} (\sigma_1 + \cdots + \sigma_n)$$

$$c = \sigma - 2D\sigma + D^2 1 1^T \sigma$$

$$\mu^T c = \mu^T \sigma - 2\mu^T D \sigma + \mu^T D^2 1 1^T \sigma$$

$$c = \sigma - 2D\sigma + \mu^2 1^T \sigma$$

<u>Sherman-Morrison formula</u>:

$$Ax = b$$

$$A = D + cd^T$$

D is diagonal

$$A^{-1} = D^{-1} - \frac{D^{-1} c d^T D^{-1}}{1 + d^T D^{-1} c}$$

$$A^{-1} A = I + D^{-1} c d^T - \frac{D^{-1} c d^T D^{-1} D}{1 + d^T D^{-1} c} - \frac{D^{-1} c d^T D^{-1} c d^T}{1 + d^T D^{-1} c}$$

$$= I + D^{-1} c d^T - \frac{D^{-1} c d^T + (d^T D^{-1} c) D^{-1} c d^T}{1 + d^T D^{-1} c}$$

$$= I + \cancel{D^{-1} c d^T} - \frac{\cancel{(1 + d^T D^{-1} c)}\, \cancel{D^{-1} c d^T}}{\cancel{1 + d^T D^{-1} c}}$$

$$c = \sigma - 2D\sigma + D^2 1 1^T \sigma$$

$$(A + uv^T)^{-1} = A^{-1} - \frac{A^{-1} u v^T A^{-1}}{1 + v^T A^{-1} u}$$

$$c = (I - 2D)\sigma + (D^2 1) 1^T \sigma$$

$$= \Big(\underbrace{\underset{A}{(I-2D)} + \underset{u}{(D^2 1)} \underset{v^T}{1^T}}_{Q} \Big) \sigma$$

$$Q^{-1} = (I - 2D)^{-1} - \frac{(I-2D)^{-1} D^2 1 1^T (I-2D)^{-1}}{1 + 1^T (I-2D)^{-1} D^2 1}$$

$$\sigma = Q^{-1} c$$

$$(I - 2D)^{-1} = \begin{pmatrix} \frac{1}{1-2\mu_1} & & & \\ & \frac{1}{1-2\mu_2} & & \\ & & \ddots & \\ & & & \frac{1}{1-2\mu_n} \end{pmatrix}$$

$$D^2 1 = \begin{pmatrix} \mu_1^2 \\ | \\ \mu_n^2 \end{pmatrix} \qquad \text{denom} = 1 + \sum_i \frac{\mu_i^2}{1-2\mu_i}$$

$$(I - 2D)^{-1} D^2 1 = \begin{pmatrix} \frac{\mu_1^2}{1-2\mu_1} \\ | \\ \frac{\mu_n^2}{1-2\mu_n} \end{pmatrix}$$

$$1^T (I - 2D)^{-1} c = \sum_i \frac{c_i}{1-2\mu_i}$$

$$(I - 2D)^{-1} c = \begin{pmatrix} \frac{c_1}{1-2\mu_1} \\ | \\ \frac{c_n}{1-2\mu_n} \end{pmatrix}$$

$$Q^{-1} c = \begin{pmatrix} \frac{c_1}{1-2\mu_1} \\ | \\ \frac{c_n}{1-2\mu_n} \end{pmatrix} - \underbrace{\frac{\sum_i \frac{c_i}{1-2\mu_i}}{1 + \sum_i \frac{\mu_i^2}{1-2\mu_i}}}_{k} \begin{pmatrix} \frac{\mu_1^2}{1-2\mu_1} \\ | \\ \frac{\mu_n^2}{1-2\mu_n} \end{pmatrix}$$

$$\sigma = \begin{pmatrix} \frac{c_1 - k\mu_1^2}{1-2\mu_1} \\ \vdots \\ \frac{c_n - k\mu_n^2}{1-2\mu_n} \end{pmatrix}$$

$$k = \frac{\sum_i \frac{c_i}{1-2\mu_i}}{1 + \sum_i \frac{\mu_i^2}{1-2\mu_i}}$$

$$k = \frac{\mathrm{sum}(c/(1-2\mu))}{1+\mathrm{sum}(\mu**2/(1-2\mu))}$$

$$\sigma = (c - k*\mu**2)/(1-2\mu)$$