# Analyzing the decline of student scores over time in self-scheduled asynchronous exams 

Binglin Chen I Matthew West I Craig Zilles

University of Illinois at Urbana-Champaign, Urbana, Illinois

## Correspondence

Binglin Chen, Department of Computer Science, University of Illinois at UrbanaChampaign, Thomas M. Siebel Center for Computer Science, 201 North Goodwin Avenue, Urbana, IL 61801.
Email: chen386@illinois.edu

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#### Abstract

Background: When students are given a choice of when to take an exam in engineering and computing courses, it has been previously observed that average exam scores generally decline over the exam period. This trend may have implications both for the design of interventions to improve student learning and for data analysis to detect collaborative cheating. Purpose/Hypothesis: We hypothesize that average exam scores decline over the exam period primarily due to self-selection effects, where weaker students tend to choose exam times later in the exam period, while stronger students are more likely to choose earlier times. Design/Method: We collected 31,673 exam records over four semesters from six undergraduate engineering and computing courses that had both synchronous exams (all students at the same time) and asynchronous exams (students choose a time). We analyzed student exam time choice and asynchronous exam scores, using synchronous exam scores in the same course as a control variable. Results: We find that students with lower scores on synchronous exams generally elect to take asynchronous exams later and that controlling for student ability (via synchronous exams) removes $70 \%$ of the decline observed in average asynchronous exam scores over the exam period but does not eliminate the downward trend with time. Conclusions: We conclude that self-selection effects are primarily responsible for exam score declines over time, that exam time selection is unlikely to be a useful target for interventions to improve performance, and that there is no evidence for widespread collaborative cheating in the dataset used in this research.


## KEYWORDS

asynchronous, undergraduate, test format (syn: Exam format), automated grading

## 1 | INTRODUCTION

There has been significant pressure on universities to increase the number of engineers graduating each year to meet workforce needs and maintain national competitiveness, and universities have responded to the call. From 2009 to 2016, the number of students awarded bachelor's degrees in engineering among major universities in the United States has increased by approximately $50 \%$ (Gibbons, 2009; Yoder, 2016), with the larger institutions growing disproportionately more quickly. A result of this growth is that in 2017, $46.6 \%$ of the bachelor's degrees in engineering were awarded by only 50 of the

305 institutions tracked by the American Society for Engineering Education (ASEE) (Yoder, 2017). This growth and concentration of students necessitate teaching techniques and tools that can maintain excellence at scale.

One aspect of teaching where scale and excellence are frequently at odds is assessment. For the past 30 years, assessing student outcomes has been recognized as the centerpiece for evaluating engineering education and hence plays a central role in both the accreditation of engineering programs via ABET and the feedback loop used to improve classes and programs (Engineering Accreditation Commission, 1998; Olds, Moskal, \& Miller, 2005; Shaeiwitz, 1996). For content-based competencies (e.g., problem-solving, interpreting data, applying knowledge/skills), the most commonly mentioned assessments in the literature are paper-and-pencil exams (Henri, Johnson, \& Nepal, 2017).

Running paper-and-pencil exams for large classes (e.g., 200+ students) presents management challenges that include requesting space, printing exams, proctoring, timely grading, and handling conflict exams (Lee, Garg, Bygrave, Mahar, \& Mishra, 2015; Muldoon, 2012; Zilles et al., 2015). These logistic burdens discourage faculty from using pedagogies that have been shown to improve student learning, such as frequent testing (Bangert-Drowns, Kulik, \& Kulik, 1991; Leeming, 2002) and mastery learning (Kulik, Kulik, \& Bangert-Drowns, 1990; Pennebaker, Gosling, \& Ferrell, 2013), and have led to the overuse of multiple-choice exams (Scouller, 1998; Stanger-Hall, 2012).

Computer-based exams have been proposed as a means of mitigating the tension between scale and excellence in assessment in engineering classes (DeMara et al., 2016; Shacham, 1998; Zilles et al., 2015). Such exams allow a broad range of questions (e.g., numeric, graphical, symbolic, programming, and drawing) to be autograded and to provide students with immediate feedback (Carrasquel, 1985; Rytkönen \& Myyry, 2014; Shacham, 1998; West, Herman, \& Zilles, 2015). Several studies have demonstrated the validity of computer-based testing across a broad range of subjects (Bodmann \& Robinson, 2004; Boevé, Meijer, Albers, Beetsma, \& Bosker, 2015; Bugbee Jr., 1996; Cagiltay \& Ozalp-Yaman, 2013; McDonald, 2002; Prisacari \& Danielson, 2017; Zandvliet \& Farragher, 1997).

Computer-based testing is particularly well suited for courses in engineering and, more generally, in the other three STEM fields of science, technology, and math. Significant amounts of the material in these courses have two important properties: (a) students' responses are well suited for digitization, and (b) these responses can be graded automatically (i.e., it is possible to write a computer program that can score a student's answer) (West, Herman, \& Zilles, 2015). This type of evaluation is especially true in analysis classes (e.g., statics, thermodynamics), but some design tasks are also amenable to this evaluation by creating tests that evaluate whether a student's solution exhibits the desired properties (e.g., whether a beam designed in a computer-aided design tool meets given stiffness and weight criteria or whether a program computes the right answer for a variety of inputs). Importantly, the use of computer-based assessment does not preclude having assessments that are not autogradeable in a course; in fact, the use of computer-based testing, where appropriate, can free faculty's/course staff's time to include more tasks that benefit from expert input (e.g., projects, lab reports, etc.) in the course (Essick, West, Silva, Herman, \& Mercier, 2016; Sanders, West, \& Herman, 2016; West, Silva-Sohn, \& Herman, 2015).

Much of the management of giving exams can be alleviated while maintaining exam security by running computer-based exams in a centralized proctored facility (Bugbee \& Bernt, 1990; DeMara et al., 2016; Rytkönen \& Myyry, 2014; Zilles, West, Mussulman, \& Bretl, 2018). To handle the varied constraints of student schedules and classes with more students than seats in a proctored computer testing center, common practice is to offer computer-based exams asynchronously (i.e., allowing students to take their exams within a given time window, usually several days) (DeMara et al., 2016; Stehlik \& Miller, 1985; Zilles et al., 2018).

Because allowing students to choose their exam time is an unusual feature in traditional university environments, it is important to study students' behavior in such settings. Chen, West, \& Zilles (2017) investigated a set of asynchronous computerized exams with randomized questions and found that students tend to choose later time slots, and exam scores generally decline throughout the exam period. Figure 1 shows these two phenomena for one asynchronous computerized exam. However, the cause of these phenomena is unclear. One possible hypothesis that deserves particular attention is that stronger students choose to take asynchronous exams whenever they feel ready, while weaker students choose to take asynchronous exams later. ${ }^{1}$ This hypothesis is consistent with the finding of a robust negative correlation between students' measured procrastination and their academic achievement (Kim \& Seo, 2015).

It is important to understand what is causing asynchronous exam scores to decline over time. If there is no separate mediating variable such as student ability, then it could mean that (a) exam time choice alone can have a detrimental impact on students' performance, and it is perhaps advisable to have some intervention in place to help students overcome nonideal exam time choices, and (b) collaborative cheating, where students who have already taken the exam share the exam questions with other students, is either not widespread or is ineffective.

In fact, faculty who consider the use of asynchronous computerized exams in their courses often question the potential for collaborative cheating resulting from the asynchronous nature of the exams. It seems initially reasonable that students taking the


FIGURE 1 Example data from one asynchronous exam (Class C3, Asynchronous Exam 4) conducted over a 4-day period. Students' raw scores are plotted against the day on which they took the exam, with the intensity of each circle being proportional to the number of students with that score on that day. The straight line is the ordinary least squares (OLS) regression line of the exam score against the day of exam, demonstrating in this case a negative correlation between the day on which the students choose to take the asynchronous exam and their score. This asynchronous exam has one of the more negative slopes in our data set, and we chose it here because the highly negative slope is easy to discern [Color figure can be viewed at wileyonlinelibrary.com]
exam on the first day would tell their friends the exam questions, giving students taking the exam later an unfair advantage. In fact, in a previous survey of undergraduate students, the most reported cheating mechanism was that they had "received answers to a quiz or test from someone who has already taken it" for face-to-face classes (Watson \& Sottile, 2010). If such cheating was effective and widespread, we would expect to see exam scores increase through the course of the exam period.

This paper, thus, aims to test the hypothesis that stronger students tend to choose to take asynchronous exams earlier than weaker students and that this is primarily responsible for the decline observed in the average score over the exam period for asynchronous exams. Previous work has attempted to address this hypothesis; however, the amount of data used in the analysis was not sufficient to draw any firm conclusion (Chen et al., 2017). In this paper, with a much larger dataset consisting of 81 asynchronous exams and 15 synchronous exams, we show that this self-selection effect indeed largely explains the observed decline in exam scores, although not all of it. The resulting slope is still negative (statistically significant with $p<.0001$ ), suggesting that unexplored factors have a larger impact on the negative slope than the benefit of collaborative cheating.

The remainder of the paper is organized as follows. In Section 2, we briefly describe relevant studies. In Section 3, we introduce the setting under which the data were collected and describe the analysis procedures. We then present the results in Section 4, discuss the implications in Section 5, and point out limitations of our study in Section 6. Finally, we provide a summary in Section 7.

## 2 | LITERATURE REVIEW

Kreiter, Peterson, Ferguson, and Elliott (2003) conducted a set of three asynchronous exams over 2 days in a clinical practice course of approximately 200 students at a Midwestern medical college. Each student was randomly assigned to take each of the three exams on one of the 2 days. Each of the three exams was kept the same over the 2-day period. Kreiter et al. (2003) reported no significant differences between students' performance on Day 1 versus Day 2 for each exam. This finding suggests that using a single test form for asynchronous exams spread over 2 days does not compromise the integrity of the exam results as long as students are randomly assigned when to take the exam. There are a few studies that describe experiments with computer-based exams in lab sections (i.e., asynchronous but at times that students chose before the start of the class) for computer science courses, but none of them has reported score trends over time (Barros, Estevens, Dias, Pais, \& Soeiro, 2003; Bennedsen \& Caspersen, 2006; Califf \& Goodwin, 2002; Jacobson, 2000).

Between 2003 and 2005, Burns, Garrett, and Childs (2007) ran 13 asynchronous computer-based exams for a microscopic anatomy course, with approximately 150 students for each exam. Each exam was identical for all students, and students were allowed to choose when to take their exams. Burns et al. (2007) found that, in general, students who choose to take exams earlier performed better than those who choose to take exams later.

Chen et al. (2017) analyzed a set of 93 asynchronous computer-based exams among nine engineering courses between 2015 and 2016 in a large public research university. In this dataset, most exams were slightly different for each student, and students were allowed to choose when to take their exams. Chen et al. reported that students tended to choose later exam time slots and that those students who elected to take the exam later performed worse on average than those who chose earlier exam time slots.

During Spring 2009, Wagner-Menghin, Preusche, and Schmidts (2013) conducted an asynchronous paper-and-pencil exam consisting of only multiple-choice questions. A total of 671 students were initially assigned to four time slots and
allowed to reschedule their exam times as they wished. By comparing the difficulty of reused items using the Rasch Model, Wagner-Menghin et al. (2013) observed that reused items became easier after their first use.

The relationship between procrastination and academic performance has been extensively studied (see Kim \& Seo, 2015, and studies cited therein). Procrastination is widespread among college students, with estimates of up to $80-90 \%$ of students engaging in it (Steel, 2007). There has been debate as to whether procrastination should be regarded as a task-specific behavior or as a personality trait that is stable across time and context (Schouwenburg, 2004), although it is now more common to adopt the latter (Kim \& Seo, 2015). While there has been significant disagreement in the literature between studies finding that procrastination does not affect academic performance (e.g., Seo, 2011; Solomon \& Rothblum, 1984) and those finding that it does (e.g., Aremu, Williams, \& Adesina, 2011; Balkis, Duru, \& Bulus, 2013), meta-analyses show that many of these differences are due to underestimates of correlations from the use of self-reported data on both procrastination and performance (Eerde, 2003; Kim \& Seo, 2015). The best overall estimates show an average correlation of $r=-.39$, $95 \%$ CI $[-0.65,-0.13]$, between measured procrastination and measured performance (Kim \& Seo, 2015).

## 3 | METHODS

## 3.1 | Data collection

The data were collected from a large R1 university in the United States during the Spring 2015, Fall 2015, Spring 2016, and Fall 2016 semesters. The asynchronous exam data were taken from exams held in the Computer-Based Testing Facility (CBTF) (Zilles et al., 2018) and administrated via the PrairieLearn system (West, Herman, \& Zilles, 2015). The synchronous exam data were provided by the corresponding instructors. A subset of these data, without the synchronous exam records, were previously used in Chen et al. (2017).

The CBTF is a computer lab with 85 seats for students and another 4 seats in a reduced-distraction environment for students registered with the disability resource center. Each of the computers is outfitted with a privacy screen that prevents test takers from reading the screens of neighboring computers, and the networking and file systems are strictly controlled (Zilles et al., 2018). During the period studied, the facility was open and proctored $10-12 \mathrm{hr}$ a day, 7 days a week to accommodate $2,000-4,000$ exams per week. Students were not permitted to take written notes, photos, or other records into or out of the exam room. At their scheduled exam time, students had their identity checked by a proctor and were randomly assigned to a computer (to deter coordinated cheating).

Exams in the CBTF were typically administered as follows (Zilles et al., 2018): Classes were assigned a 3-5-day period for the students to take an exam depending on the class size; longer exam periods were used during finals week. Students were free to reserve any time during the exam period, provided that there were slots available at that time. Sign-ups for exams typically started 2 weeks before the exam period began. Generally, the periods of exams from different classes overlapped one another, and the CBTF was almost always running several distinct exams concurrently.

PrairieLearn is an online problem-posing system that permits the specification of automatic item generators (AIGs) (Attali, 2018), each of which is capable of generating a range of parameterized problem instances (West, Herman, \& Zilles, 2015). A variety of problem types can be specified, including but not limited to numeric, graphical, symbolic, programming, and drawing problems. For exams, PrairieLearn selected random problem generators from a pool of available generators and randomly generated problem instances from those generators to meet instructor-defined coverage and difficulty criteria. Students sitting next to each other in the CBTF were typically taking exams from different courses, but even if they were taking the same exam, they generally had different sets of parameterized questions or the same set of questions with different parameters. PrairieLearn also supports allowing students to have multiple attempts at each question with a partial-credit schedule controlled on a per-question basis.

For each student taking an exam in the CBTF, PrairieLearn logged all the submissions the student made during the exam period and calculated and stored the final score based on the instructor's multiple-attempts scoring scheme.

## 3.2 | Data description and preprocessing

The courses studied are drawn from the introductory sequences in mechanical engineering (statics, dynamics, and strength of materials) and computer science (intro to programming, computer organization, and system programming). The courses in mechanical engineering are primarily focused on engineering sciences, while the courses in computer science involve a combination of computing science and code writing. As these courses are introductory, course material is primarily fixed with little
variation even when different instructors teach them. The asynchronous exam material is usually first developed by a single instructor over several semesters and then augmented by other instructors.

For each class in each semester, we obtained the information of all of the asynchronous exams in the form of class ID, exam ID, start date, and end date. The class ID is a unique identifier to differentiate between each class in each semester. The exam ID is a unique identifier for each exam. The start date is the first calendar day when students can take the exam. The end date is the last calendar day when students can take the exam. We refer to the time period defined by the start date and the end date as the exam period. The synchronous exam information is in the form of (class ID, exam ID), where class ID and exam ID are the same as in the asynchronous case.

With the approval of the institutional review board, we obtained all of the students' asynchronous exam records in the CBTF, as well as their synchronous exam records outside the CBTF, for each class in each semester. Each asynchronous exam record has the form (exam ID, student ID, score, date, day, hour). The exam ID is the same as defined above. The student ID is a unique identifier for a student regardless of class. The score is a real number ranging from 0 to 100 . The date is the calendar date when the student took the exam. The day is an integer ranging from one to the length of the exam period. ${ }^{2}$ The hour is an integer ranging from 0 to 23 . Each synchronous exam record has the form (exam ID, student ID, score), where they are the same as in the asynchronous case.

Given the raw asynchronous exam data, we used the following filters:

1. We excluded all optional second-chance asynchronous exams that allowed students to replace part or all of an earlier asynchronous exam score by taking a second equivalent asynchronous exam at a later date.
2. We excluded students who took less than $50 \%$ of the nonsecond-chance asynchronous exams to avoid including course staff members engaged in exam checking and students who dropped early in the semester.
3. We excluded students who did not have the corresponding synchronous exam records.
4. We excluded asynchronous exam records that were outside the corresponding exam periods. ${ }^{3}$
5. We excluded asynchronous exams whose score distribution's kurtosis was more than 10 . These exams had an unusually high number of scores that were greater than several standard deviations away from the mean.

The first three points primarily aim to filter out data irrelevant to the analysis. The fourth filter eliminates those asynchronous exam records that are often outliers from the analysis as most students take exams within the exam period. The fifth filter eliminates asynchronous exams that have large deviations from the mean, which could have unstable effects on the regression coefficients. We also applied this filter to the synchronous exam data, and none of the synchronous exams was excluded.

We examined the statistical characteristics of exam score distributions after the above filtering for both synchronous and asynchronous exam scores and found that they were similar to each other and match the characteristics of typical exam score distributions reported in the literature. See Appendix A for more details.

The filtering resulted in 26,139 exam records from 81 asynchronous exams and 5,534 exam records from 15 synchronous exams. A summary of the data is shown in Table 1. For courses with only one synchronous exam, these exams were either the final exam or a midterm toward the end of the semester. For the course with three synchronous exams, these exams were midterms and final.

Unfortunately, the data collected do not contain demographic information for the students; thus, our analysis focuses on the population as a whole. As an estimate of the demographic composition of the students in the data, we reported the demographic information of undergraduates who graduated with degrees in each discipline during the calendar year of 2018 in Table 2.

To analyze different exams with different score distributions together, we standardized all of the exam scores to $z$ scores on a per-exam basis. Essentially, the standardized score measures how many standard deviations a particular student's score is away from the mean. We refer to the exam scores after standardization as the standardized score. In addition, for each class and semester, we define the synchronous score to be the average of all of the standardized scores for synchronous exams. For example, if a class in a particular semester has three synchronous exams and a student's standardized score for the three are $-0.5,1.0$, and 1.0 , then the synchronous score of the student is 0.5 for that particular class in that semester.

Because asynchronous exams had different exam period lengths and the CBTF operation hours changed slightly from semester to semester, scaling was necessary for the analysis to be meaningful. Specifically, we scaled the day of the exam period to the range $[0,1]$, where the first day of the exam period is represented by 0 and the last day of the exam period is represented by 1 . We scaled the hour of day to the range $[0,1]$, where the hour of the first asynchronous exam of each day is

TABLE 1 Summary information of the data used in the analysis

| Course <br> and <br> semester $^{\text {a }}$ | Discipline ${ }^{\text {b }}$ | DFW rate (\%) | Number of students | Number of asynchronous exams included ${ }^{\text {c }}$ | Number of asynchronous exams excluded | Number of asynchronous exam records excluded ${ }^{\text {d }}$ | Number of synchronous exams |
| :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: |
| Class A2 | ME | 12.6 | 566 | 5 | 1 | 22 | 3 |
| Class B2 | ME | 10.2 | 230 | 7 | 0 | 0 | 1 |
| Class B3 | ME | 11.6 | 345 | 6 | 0 | 5 | 1 |
| Class B4 | ME | 12.8 | 181 | 7 | 0 | 3 | 1 |
| Class C2 | CS | 20.0 | 173 | 5 | 1 | 8 | 1 |
| Class C3 | CS | 17.9 | 325 | 4 | 3 | 0 | 1 |
| Class C4 | CS | 18.8 | 292 | 3 | 4 | 1 | 1 |
| Class D1 | ME | 9.5 | 477 | 2 | 0 | 0 | 1 |
| Class E1 | CS | 18.8 | 324 | 7 | 1 | 1 | 1 |
| Class E2 | CS | 11.8 | 352 | 8 | 0 | 24 | 1 |
| Class E3 | CS | 14.9 | 187 | 9 | 0 | 6 | 1 |
| Class E4 | CS | 13.0 | 369 | 8 | 1 | 0 | 1 |
| Class F4 | CS | 2.0 | 581 | 10 | 0 | 68 | 1 |
| Total |  |  |  | 81 | 11 | 138 | 15 |

Note: This table includes only nonsecond-chance exams. Some courses started using the CBTF/PrairieLearn environment in later semesters, and some courses stopped running synchronous exams in later semesters. There is only one column for synchronous exams as none of them is excluded.
${ }^{\mathrm{a}}$ Each course is indicated by a letter (A-F) and a number for the semester ( $1=$ Spring 2015, $2=$ Fall 2015, $3=$ Spring 2016, $4=$ Fall 2016 $)$.
${ }^{\mathrm{b}} \mathrm{CS}$ stands for computer science, and ME stands for mechanical engineering.
${ }^{\text {c }}$ By "exams," we mean a unique exam available to all the students of the course.
${ }^{\text {d }}$ By "exam records," we mean the record of an individual student taking an exam. See Section 3.2 for more details.

TABLE 2 Demographic information of undergraduates who graduated with degrees in each discipline during the calendar year 2018 in percentages

|  | Mechanical engineering | Computer science |
| :--- | :---: | :---: |
| Female | 15.1 | 21.1 |
| International | 20.9 | 33.8 |
| Hispanic | 7.5 | 2.1 |
| Asian American | 15.6 | 35.1 |
| Black | 0.9 | 0.4 |
| White | 50.7 | 26.9 |
| Other $^{\mathrm{a}}$ | 4.4 | 1.7 |

${ }^{\text {a }}$ People who did not select Hispanic, Asian American, Black, or White.
represented by 0 , and the hour of the last asynchronous exam of each day is represented by 1 . We refer to the day of exam period and hour of day after the scaling as scaled day and scaled hour, respectively.

## 3.3 | Analysis

Our analysis consists of four parts. We first show the general trend of students' exam time choices in Section 4.1. We provide the distribution of exam time choices for a typical asynchronous exam, as well as the overall trend. We then disaggregate students into three groups based on their synchronous score to examine if there is any difference among students of different levels of ability.

The second part of the analysis consists of examining the distribution of correlation coefficients between four pairs of measures. The first pair is between standardized scores of an asynchronous exam and the corresponding synchronous score. The purpose of this pair is to determine how well asynchronous exam scores correlate with synchronous exam scores. To determine whether the correlations between the first pair are reasonable, the ideal comparison would be the correlation
coefficients between scores of different synchronous exams for each class and each semester. Unfortunately, most classes had only one synchronous exam as Table 1 shows. Instead, as the second pair, we compute correlations between the scores of asynchronous exams that belong to the same class and same semester. The third pair is between the scaled day of an asynchronous exam and the corresponding synchronous score. It is important to examine this pair because our hypothesis suggests that it should be negative for most asynchronous exams. For comparison, we also study the correlation coefficients between standardized asynchronous score and scaled day of asynchronous exams as the fourth pair. We present the result of this analysis in Section 4.2.

Our third analysis quantifies the effect on their performance when students choose to take asynchronous exams. We use the same analysis techniques as in the previous work (Chen et al., 2017). Specifically, we regress the standardized score against scaled day and scaled hour as follows:

$$
\begin{equation*}
z_{i k}=\alpha_{k}+\beta_{k} d_{i k}+\gamma_{k} h_{i k} \tag{1}
\end{equation*}
$$

where $z_{i k}, d_{i k}$, and $h_{i k}$ are observed values from the data defined as follows:
$z_{i k}$ is the standardized score of student $i$ on asynchronous exam $k$
$d_{i k}$ is the scaled day of student $i$ taking asynchronous exam $k$
$h_{i k}$ is the scaled hour of student $i$ taking asynchronous exam $k$
$\alpha_{k}, \beta_{k}$, and $\gamma_{k}$ are the regressors that we want to calculate, defined as follows:
$\alpha_{k}$ is the intercept for asynchronous exam $k$
$\beta_{k}$ is the coefficient that characterizes the effect of scaled day on scores for asynchronous exam $k$
$\gamma_{k}$ is the coefficient that characterizes the effect of scaled hour on scores for asynchronous exam $k$.
The slope $\beta$ is expressed in units of standard deviation per exam period, so a value of $\beta=-0.5$ would mean, roughly speaking, that the student asynchronous exam scores decline by half of a standard deviation from the first day to the last day of the asynchronous exam. The slope $\gamma$ is expressed in units of standard deviation per day, so a value of $\gamma=-0.1$ means, roughly speaking, that student asynchronous exam scores decline by one tenth of a standard deviation from the first hour of each day to the last hour of each day. We refer to this regression as the uncontrolled regression and report the results in Section 4.3.

As there are multiple independent variables in the regression, we report the maximum variance inflation factor (VIF) (Kutner, Nachtsheim, \& Neter, 2004) for each of the relevant regressors to examine if multicollinearity can undermine the interpretability of the coefficients. The VIF for a particular regressor $\delta_{k}$ is defined as:

$$
\begin{equation*}
\mathrm{VIF}_{\delta_{k}}=\frac{1}{1-R_{\delta_{k}}^{2}} \tag{2}
\end{equation*}
$$

where $R_{\delta_{k}}^{2}$ is the coefficient of determination (correlation coefficient squared) for the regression of $\delta_{k}$ on the other regressors. $\mathrm{VIF}_{\delta_{k}}$ is a multiplicative term in the calculation of $\hat{\sigma}_{\delta_{k}}^{2}$, which essentially quantifies how much inflation in the observed variance of $\delta_{k}$ is contributed by correlation among regressors (O'Brien, 2007). The lower bound of $\mathrm{VIF}_{\delta_{k}}$ is 1 when $R_{\delta_{k}}^{2}=0$, and there is no upper bound. A large VIF for a regressor indicates that the observed variance of the coefficient of the regressor is inflated substantially, and the resulting confidence interval of the coefficient of the regressor is much wider than when there is little correlation among regressors.

In the fourth analysis, we add one additional regressor, $\delta_{k}$, in the uncontrolled regression:

$$
\begin{equation*}
z_{i k}=\alpha_{k}+\beta_{k} d_{i k}+\gamma_{k} h_{i k}+\delta_{k} c_{i k} \tag{3}
\end{equation*}
$$

where $c_{i k}$ is the observed synchronous score of student $i$ corresponding to asynchronous exam $k$, and $\delta_{k}$ is the coefficient that quantifies the effect of synchronous score. The slope $\delta$ is expressed in units of standard deviation of an asynchronous exam per standard deviation of a synchronous exam, so a value of $\delta=0.75$ means, roughly speaking, that a student whose synchronous score is 2 standard deviations higher than the average will obtain an asynchronous score that is 1.5 standard deviations higher than the average. This process of adding potential confounding factors to the regression formula to see if the coefficient
of interest changes substantially is a standard procedure to verify confounding factors (Kleinbaum, Kupper, \& Morgenstern, 1982; Kleinbaum, Kupper, Muller, \& Nizam, 1998). We refer to this regression as the controlled regression and report the results in Section 4.4.

## 4 | RESULTS

## 4.1 | Students' exam time choices

We observed that more students chose to take asynchronous exams on later days of the exam period and at later hours of each day (especially on the last day) even though they were allowed to choose when to take their asynchronous exams up to 2 weeks before the exam period began. We plotted an example of the day and hour distributions for one exam in Figure 2. As this figure shows, a large majority of the students took the exam on the last day. While the hour distribution of the first few days are somewhat spread out, the hour distribution of the last day is biased toward later hours, especially the last hour.

We plotted the distribution of the student asynchronous exam records for each asynchronous exam with respect to scaled day in Figure 3. As the figure suggests, students overwhelmingly choose to take asynchronous exams toward the end of the exam period. The few segments that drop at the end correspond to final exams where students may have wanted to leave campus early.

To have a better understanding of how students' ability relates to their exam time choices for asynchronous exams, we separated students into "High," "Mid," and "Low" equal-sized groups on a per-class basis based on their synchronous scores and plotted the distribution of student exam records of all asynchronous exams aggregated for each group in Figure 4. We aggregated different asynchronous exams by binning points on scaled day to intervals $[0,0.25),[0.25,0.50),[0.50,0.75),[0.75$, $1.00),[1.00,1.00]$, and averaged their $y$-axis values. We then plotted the averaged value on the left side of each interval. As the results in the figure show, "High" students' exam time choice is the most evenly distributed, and the distributions are concentrated more at the end of the exam period as we move from "High" to "Low."

We also plotted the distribution of student asynchronous exam records for each asynchronous exam on the last day with respect to scaled hour in Figure 5. As this figure shows, there is a bias toward the later hours on the last day. These figures indicate that the example in Figure 2 is indeed representative.

## 4.2 | Correlation analysis

We plotted a series of distributions of correlation coefficients and their significance in Figure 6. Specifically, we plotted the number of significant $(p<.05)$ correlations in light green and the number of nonsignificant ones in dark blue as a stacked bar.

The first subplot shows the distribution of correlation coefficients between the synchronous score and the standardized score of asynchronous exams. Each correlation coefficient is calculated using the standardized score of one asynchronous exam and the corresponding synchronous score. As the figure shows, all pairs are positively correlated, and the coefficients center around $0.4-0.5$, mean $r=.432,95 \%$ CI [0.401, 0.462]. Almost all of them are significant at the $p<.05$ level. As a reference, the distribution of correlation coefficients between the scores of asynchronous exams that belong to the same class and same semester is plotted in the second subplot of Figure 6. As the subplot shows, all of the correlation coefficients are positive and centered around $0.3-0.4$, mean $r=.330,95 \%$ CI [0.313, 0.346]. Most of them are significant at the $p<.05$ level. We take the result of this comparison as positive evidence that the correlations between synchronous scores and standardized scores of asynchronous exams are reasonable.


FIGURE 2 Example of exam record distributions during the exam period for one exam. Students choose to take exams on later days, especially the last day, and in later hours of the last day [Color figure can be viewed at wileyonlinelibrary.com]


FIGURE 3 Distribution of student exam records over the exam period for all asynchronous exams. Each series of connected line segments represents the distribution for a single asynchronous exam. The horizontal axis shows the scaled day, with 0 representing the first day of each exam and 1 representing the last day. We scaled the vertical axis values so that all line segments would overlap with the dashed line if student exam records were uniformly distributed over the exam period [Color figure can be viewed at wileyonlinelibrary.com]

FIGURE 4 Distribution of student exam records over the exam period for all asynchronous exams for "High," "Mid," and "Low" students separately. Axes are the same as in Figure 3 [Color figure can be viewed at wileyonlinelibrary.com]

FIGURE 5 Distribution of student exam records over the operation hours of the last day for all of the asynchronous exams. Each series of connected line segments represents the distribution for a single asynchronous exam. The horizontal axis shows the scaled hour, with 0 representing the hour of the first exam record on the last day of each exam and 1 representing the hour of the last exam record on the last day. We scaled the vertical axis values so that all the line segments would overlap with the dashed line if student exam records were uniformly distributed over the operation hours of the last day of each exam [Color figure can be viewed at wileyonlinelibrary.com]

The distribution of correlation coefficients between synchronous scores and scaled days of asynchronous exams is plotted in the third subplot of Figure 6. As the subplot suggests, they are mostly negatively correlated at around -0.3 to -0.2 , mean $r=-.215,95 \%$ CI [ $-0.233,-0.198]$. A few of the correlation coefficients are actually positive but relatively close to 0 . Most of the correlation coefficients are significant ( $p<.05$ ) except those near 0 . This result is consistent with our hypothesis that weaker students choose to take asynchronous exams on later days of the exam period. For comparison, we plotted the distribution of correlation coefficients between standardized asynchronous scores and scaled days of asynchronous exams on the last subplot of Figure 6 . Most correlation coefficients are significant ( $p<.05$ ), centering around -.2 to -.1 , mean $r=-.128,95 \%$ CI $[-0.149,-0.107]$, and a slightly higher number of correlation coefficients are positive compared to the previous case. Overall, all the observations in the figure are consistent with our hypothesis and previous observations.


FIGURE 6 Distribution of correlation coefficients between synchronous score (sync), standardized score of asynchronous exams (async), and scaled day of asynchronous exams (day). Note that the third subplot involves synchronous scores correlated with the scaled day of the corresponding asynchronous exams. We plotted the total number of correlation coefficients in each bin in dark blue and the number of significant $(p<.05)$ correlation coefficients in each bin in light green [Color figure can be viewed at wileyonlinelibrary.com]


FIGURE 7 Joint forest plot that compares the slopes $\beta_{k}$ of standardized asynchronous score versus scaled day under uncontrolled (top) and controlled (bottom) conditions. Each circle represents the slope of one asynchronous exam, and they are grouped by course and semester as shown on the left. The area of each circle is proportional to the weight $w_{k}=1 / v_{k}$ of the corresponding exam in the meta-analysis, and the horizontal error bar is the $95 \%$ confidence interval for the slope. The diamond at the bottom of the upper part of the figure represents the aggregate population slope, $\beta=-0.390,95 \%$ CI $[-0.453,-0.328]$, for all of the exams under the uncontrolled analysis, and the diamond at the bottom of the lower part of the figure represents the aggregate population slope, $\beta=-0.115,95 \% \mathrm{CI}[-0.168,-0.063]$, for all of the exams under the controlled analysis. The width of the diamonds specifies the $95 \%$ confidence intervals of the estimates. The two-tailed significance levels of the slopes away from zero are shown on the right of the figure as a number of stars [Color figure can be viewed at wileyonlinelibrary.com]

## 4.3 | Uncontrolled regression

The maximum VIF for both $\beta_{k}$ and $\gamma_{k}$ among all the asynchronous exams is 1.295 . These VIFs are reasonably small, and thus, multicollinearity is not a concern.

We visualize each asynchronous exam's $\beta_{k}$ and $\gamma_{k}$ with its $95 \%$ confidence intervals on a pair of forest plots in the upper parts of Figures 7 and 8, respectively. A forest plot is a standard meta-analysis visualization tool (Cooper, Hedges, \& Valentine, 2009) that shows effect sizes for many different studies together with their confidence intervals (horizontal bars) and an indicator of study reliability (area of circles).

The two-tailed significance levels ( $p$ values) for the slopes being nonzero are shown on the right of the figures. For the effect of scaled day $\left(\beta_{k}\right)$, none $(0 \%)$ of the slopes are statistically significantly positive ( $p<.05$ ); $6(7 \%)$ of the slopes are nonsignificantly positive ( $p>.05$ ); $46(57 \%)$ of the slopes are statistically significantly negative, and $29(36 \%)$ are nonsignificantly negative. For the effect of scaled hour $\left(\gamma_{k}\right), 2(2 \%)$ of them are significantly positive; $17(21 \%)$ are nonsignificantly positive; $18(22 \%)$ are significantly negative; and $44(54 \%)$ are nonsignificantly negative. To obtain an aggregated measure of $\beta$ and $\gamma$, we adopted the standard meta-analysis techniques described in Cooper et al. (2009). Although there is no clear consensus in


Slope $\gamma$ (change in score with hour of exam) / standard deviation per exam period of a day
FIGURE 8 Joint forest plot that compares the slopes $\gamma_{k}$ of standardized asynchronous score versus scaled hour under uncontrolled (top) and controlled (bottom) conditions. The aggregate population slope under the uncontrolled analysis is $\gamma=-0.181,95 \% \mathrm{CI}[-0.240,-0.121]$, while the aggregate population slope under the controlled analysis is $\gamma=0.019,95 \% \mathrm{CI}[-0.033,0.071]$. See Figure 7 for a description of the figure format [Color figure can be viewed at wileyonlinelibrary.com]


FIGURE 9 Normal probability plots for the slopes $\beta_{k}, \gamma_{k}$, and $\delta_{k}$ from all exams under uncontrolled and controlled settings. These plots show that the slopes are approximately normally distributed [Color figure can be viewed at wileyonlinelibrary.com]
the meta-analysis community on how to combine regression slopes in the general case (Cooper, 2016), previous work (Becker \& Wu, 2007; Cooper, 2016) suggests that, under the condition when both the dependent and independent variables are measured similarly across studies, the regression slopes can be safely combined by treating them as a simple effect. This is the approach that we adopted. Details about these techniques are included in Appendix B. One important assumption of these is that the data are normally distributed. We plotted the normal probability plots of $\beta_{k}$ and $\gamma_{k}$ for the uncontrolled regression at the top of Figure 9. As the figure suggests, they are both approximately normally distributed. Using meta-analysis techniques (see Appendix B for more details), we obtained aggregate $\beta=-.390,95 \%$ CI $[-0.453,-0.328]$, which is significantly negative $(p<.0001)$, and aggregate $\gamma=-.181$, $95 \%$ CI $[-0.240,-0.121]$, which is also significantly negative $(p<.0001)$. The aggregates $\beta$ and $\gamma$ are plotted as diamonds at the bottom of the upper parts of Figures 7 and 8 , respectively. We have also computed the $R^{2}$ for each asynchronous exam, and the average $R^{2}$ for the uncontrolled regression is 0.035 .

To explore the impact of filtering students who have dropped the course, we calculated the aggregates $\beta$ and $\gamma$ including these students. In this case, $\beta=-0.397,95 \% \mathrm{CI}[-0.462,-0.332]$, and $\gamma=-0.193,95 \% \mathrm{CI}[-0.256,-0.130]$. Both values become slightly more negative than those calculated without students who have dropped the course. However, each point estimate is still well within the respective original confidence interval.

## 4.4 | Controlled regression

The maximum VIFs for $\beta_{k}, \gamma_{k}$, and $\delta_{k}$ are $1.366,1.335$, and 1.237 , respectively. These VIFs are again reasonably small, and thus, multicollinearity is not a concern. The distributions of $\beta_{k}$ and $\gamma_{k}$ are shown in the lower part of Figures 7 and 8 , respectively, where each $\beta_{k}$ and $\gamma_{k}$ is plotted with its $95 \%$ confidence interval.

We again show the two-tailed significance levels ( $p$ values) for the slopes being nonzero on the right of the figures. For the effect of scaled day $\left(\beta_{k}\right), 5(6 \%)$ of them are statistically significantly positive ( $p<.05$ ); $17(21 \%)$ are nonsignificantly positive ( $p>.05$ ); $13(16 \%)$ are significantly negative; and $46(57 \%)$ are nonsignificantly negative. For the effect of scaled hour $\left(\gamma_{k}\right)$, $6(7 \%)$ are significantly positive; $38(47 \%)$ are nonsignificantly positive; $4(5 \%)$ are significantly negative; and 33 (41\%) are nonsignificantly negative. We plotted the normal probability plots of $\beta_{k}, \gamma_{k}$, and $\delta_{k}$ for the controlled regression at the bottom of Figure 9. As the figure suggests, they are approximately normally distributed. The same methodology as in the uncontrolled
regression was used to calculate the aggregates $\beta$ and $\gamma$, and we obtained $\beta=-.115,95 \% \mathrm{CI}[-0.168,-0.063]$, which is significantly negative $(p<.0001)$, and $\gamma=.019,95 \%$ CI $[-0.033,0.071]$, which is nonsignificantly positive ( $p>.1$ ). The aggregates $\beta$ and $\gamma$ are plotted as diamonds at the bottom of the lower parts of Figures 7 and 8 , respectively. We also computed the $R^{2}$ for each asynchronous exam, and the average $R^{2}$ for the controlled regression is 0.217 .

## 5 | DISCUSSION

The purpose of this paper was to explore the previously observed phenomenon where, when given a choice of when to take an exam, many students choose to take it toward the end of the exam period and, on average, perform worse than students who choose earlier times (Chen et al., 2017). Our hypothesis for the cause of this phenomenon is that weaker students tend to put off the exam, while stronger students tend to take the exam over a more uniform distribution of times. That is, we hypothesize that weaker students procrastinate more.

To test our hypothesis, we investigated data from courses that have run both synchronous exams (all students take the exam at the same time) and asynchronous exams (students can choose when to take the exam within a short time period, usually $3-5$ days) in the same semester. The synchronous exams were typically midterms and finals, which were weighted more heavily in the course grades than asynchronous exams. The synchronous exams typically occurred chronologically in the middle or after the asynchronous exams. We found that students' choices of exam time negatively correlate with their scores on synchronous exams, $r=-.215,95 \%$ CI $[-0.233,-0.198]$, meaning that students with lower scores on synchronous exams tend to choose to take asynchronous exams later. This result is consistent with the best estimates of the correlation between measured procrastination and measured academic achievement of $r=-0.39,95 \%$ CI [-0.65, -0.13] (Kim \& Seo, 2015), suggesting that student ability may in fact be a mediating variable between exam time choice and exam performance.

By using students' scores on synchronous exams as a control (Equation (3)), we found that the magnitude of the decline observed in asynchronous exam scores throughout the exam period reduces considerably. As Figures 7 and 8 show, when synchronous score is added to the regression, both $\beta$ and $\gamma$ shift substantially to the right. Specifically, $\beta$ moves from -0.390 to -0.115 , which corresponds to about a $70 \%$ reduction in its value while remaining significantly below zero; $\gamma$ changes from -0.181 to 0.019 , which is a change away from significantly negative to nonsignificance. These drastic changes suggest that the decline observed in asynchronous exam score over the exam period can be largely attributed to the confounding factor of synchronous exam score. In other words, students' ability, as measured by synchronous exam scores, explains the majority of the declining trend. However, the coefficient corresponding to the decline of student scores over time is still statistically significantly negative even when students' ability is taken into account, suggesting that there are other factors causing students' scores to decline over time and that there are no countervailing effects (such as widespread collaborative cheating) large enough to cause average scores to increase over time.

While our results suggest that students' performance in synchronous exams plays an important role in choosing exam times in the asynchronous setting, our data do not reveal why weaker students choose these later time slots. One hypothesis is that they choose later times because they give these students the most time for studying, although a longer study time does not necessarily result in better performance (Kember, Jamieson, Pomfret, \& Wong, 1995; Plant, Ericsson, Hill, \& Asberg, 2005). Another hypothesis is that they procrastinate and end up with an equal amount of study time as for a synchronous exam because it is well known that procrastination negatively correlates with academic performance (Richardson, Abraham, \& Bond, 2012). Either way, our results suggest that interventions focused on directly preventing students from scheduling their exams later will not necessarily improve students' performance as their choice of exam time is closely related to their performance on synchronous exams.

Importantly, our study also provides two pieces of evidence supporting the use of asynchronous computerized exams as a viable alternative for synchronous paper-and-pencil exams. The first is the positive correlations observed between asynchronous exam scores and synchronous exam scores. The second is that the decline of scores over time is not fully neutralized even when synchronous scores are controlled for, suggesting that widespread collaborative cheating is not present in the asynchronous setting. These two observations support the hypothesis that asynchronous computerized exams with proper proctoring and randomization can achieve an integrity similar to synchronous paper-and-pencil exams.

Our study suggests that the particular setup of the CBTF and precautions taken for CBTF exams are sufficient for successful asynchronous computerized exams. To summarize, the CBTF is a normal computer lab converted for testing purposes, where its file system and network are restricted. The CBTF is proctored while exams are running. Students are not allowed to take notes in or out of the CBTF. Exam questions are randomly selected and parameterized. We encourage the adoption of similar strategies by institutions that wish to have their own asynchronous computerized environments.

There are a few obvious benefits of asynchronous randomized computerized exams. The computerized format allows questions with more sophisticated formats to be automatically graded, thus allowing class sizes to scale and reducing grading time. The randomized questions reduce exam development time in the long run as items built with randomization can be reused from semester to semester. The asynchronous scheduling virtually eliminates the need for conflict exams and simplifies the handling of exceptions such as student illness. These three benefits facilitate frequent testing for large classes, thus enabling the use of the well-known testing effect at scale (Phelps, 2012; Roediger III \& Karpicke, 2006) and potentially leading to improved student-learning outcomes (Nip, Gunter, Herman, Morphew, \& West, 2018).

## 6 | LIMITATIONS

As the study in this paper used existing data from real courses, it has a number of limitations. First, we did not have access to demographic information corresponding to individual student records, so we were unable to test for demographic correlates with student behavior or results. In future work, it will be important to explore whether specific subgroups of students exhibit different outcomes in the asynchronous computerized exam system. It is easy to imagine that there could be multiple complex and interacting effects, such as asynchronous exams benefiting nontraditional students with family or work obligations or asynchronous exams disadvantaging nontraditional students without well-established study habits. If we had individual student demographic information, we could examine this by repeating our analysis disaggregated by subgroup.

Second, it is unclear the extent to which our results can be generalized beyond the setting of engineering courses at a large R1 university in the United States. The student body in this study is likely only representative of similarly situated universities and of similar engineering, and perhaps STEM, courses. Compared to national numbers for the engineering student population, the population in the university where the data were collected has slightly fewer females ( $19.5 \% / 21.3 \%$ ) and fewer White ( $41.5 \% / 62.3 \%$ ), Black ( $1.8 \% / 4.1 \%$ ), and Hispanic ( $5.8 \% / 11.1 \%$ ) students but more American Asian $(22.1 \% / 14.6 \%)$ and foreign $(25.6 \% / 3.8 \%)$ students (Yoder, 2017). It would be very interesting to compare data from other environments.

Third, our data were limited in the number of synchronous exam controls we had for each course. Most of the courses for which we had data offered only a single synchronous exam during the semester. While we have no reason to believe that this would cause a consistent bias in using the synchronous exam as an estimate of student ability in the course, it is likely that this exam does not measure the exact same skills as the asynchronous exams. Thus, our analysis may underestimate the extent to which the decline of student scores over the exam period is due to the correlation between student ability and exam time selection. Access to other measures of student ability would offer more control that could improve our analysis.

Fourth, we did not have detailed information about the exams in each course. As a result, we were unable to investigate the effect of different question types (e.g., multiple choice vs. writing code), different exam purposes (e.g., primarily low-stakes formative feedback vs. high-stakes summative assessments), or the amount of variation in questions given to different students. Future work with access to per-exam and per-question details could help to elucidate the extent to which these and other factors alter the effects analyzed in this work.

## 7 | CONCLUSION

We examined 26,139 asynchronous exam records from 81 asynchronous exams and 5,534 exam records from 15 synchronous exams, all gathered over four semesters from six engineering and computer science courses. We tested the hypothesis that the observed score decline, where students' average performance drops over the exam period in asynchronous exams, can be attributed to weaker students electing to take exams later in the exam period. We found that students' choices of exam time negatively correlate with their scores on synchronous exams, meaning that students with lower scores on synchronous exams tend to choose to take asynchronous exams later. Furthermore, we found that students' performance on synchronous exams explains approximately $70 \%$ of the observed decline in student scores over the exam period, where the observed decline is characterized by $\beta$ in the uncontrolled regression (Equation (1)). This observation indicates that the majority of the observed decline in student scores over the exam period is due to students with different levels of ability choosing to schedule their exams earlier or later in the exam period.

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## ENDNOTES

${ }^{1}$ By stronger/weaker students, we mean students who are observed to do well/poorly in synchronous exams.
${ }^{2}$ The exam period length varies across exams. It is generally $3-5$ days and up to 8 days for final exams.
${ }^{3}$ In exceptional circumstances such as long-term illness, students take an asynchronous exam outside the normal exam period.

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## AUTHOR BIOGRAPHIES

Binglin Chen is a PhD student in Computer Science at the University of Illinois at Urbana-Champaign, Urbana, IL 61801; chen386@illinois.edu

Matthew West is an Associate Professor of Mechanical Engineering at the University of Illinois at Urbana-Champaign, Urbana, IL 61801; mwest@illinois.edu

Craig Zilles is an Associate Professor of Computer Science at the University of Illinois at Urbana-Champaign, Urbana, IL 61801; zilles@illinois.edu

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## APPENDIX A: CHARACTERISTICS OF EXAM SCORE DISTRIBUTIONS

We will show that the exam score distributions of asynchronous exams do not differ much from that of synchronous exams after the filtering. This is important as discrepancies between the two might indicate something unusual is going on in asynchronous exams. We use skewness and kurtosis to describe the distributions in addition to the mean of the scores as in the literature (Cook, 1959; Ho \& Yu, 2015; Lord, 1955). The skewness is a measure of the asymmetry of a distribution where negative skewness means a longer left tail and positive skewness means a longer right tail. The kurtosis is a measure of "tailedness" of a distribution where larger kurtosis indicates there are more values at the tail, and normal distributions have a kurtosis value of 3 . The squared skewness plus one is a lower bound of kurtosis because knowing a distribution is skewed already puts some constraints on the tailedness of the distribution. We plotted the overall distribution of kurtosis versus skewness of asynchronous exams in the top left subplot of Figure A1. To give a concrete sense of what score distributions lead to the different skewness/kurtosis, we plotted score distributions of three selected asynchronous exams around the scatter plot.

We plotted the mean, skewness, and kurtosis of asynchronous exams and synchronous exams side by side in Figure A2 to compare them. As the figure shows, there is no distinctive difference between the two types of exams, and there are two noticeable trends in the distributions: (a) exams with mean above $50 \%$ tend to have negative skewness, and (b) exams with near symmetric distributions tend to have negative excess kurtosis compared to normal distribution, which has kurtosis 3. These two trends for exams have been observed since the middle of the last century (Cook, 1959; Ho \& Yu, 2015; Lord, 1955) and indicate that these exams have typical score distributions.

To help understand what kind of asynchronous exams are excluded, we plotted the exam score distributions of two excluded asynchronous exams due to large kurtosis in Figure A3. The main feature of these excluded asynchronous exams is the presence of exam scores that are far from the mean in terms of number of standard deviations.


FIGUREA1 The subplot on the top left is the distribution of kurtosis versus skewness of all the asynchronous exams after the filtering. Each data point in the scatter plot represents a single asynchronous exam. The dashed curve is the lower bound for kurtosis in terms of skewness. The other three subplots are the exam score distributions of the highlighted asynchronous exams in the skewnesskurtosis subplot [Color figure can be viewed at wileyonlinelibrary.com]


FIGUREA2 Summary statistics for both the asynchronous exams and the synchronous exams. Each data point represents one exam. The dashed curves in the bottom plots are the lower bound for kurtosis in terms of skewness [Color figure can be viewed at wileyonlinelibrary.com]


FIGUREA3 Examples of score distributions of excluded asynchronous exams whose kurtosis is larger than 10. These asynchronous exams generally have scores that are far from the mean in terms of standard deviation [Color figure can be viewed at wileyonlinelibrary.com]

## APPENDIX B: META-ANALYSIS

Meta-analysis techniques essentially deal with cases where it is not desirable to directly average effect sizes to compute the mean. Specifically, the task we are trying to solve is: Given a set of $k$ observed effect sizes $T_{1}, \ldots, T_{k}$, each with their unknown true effect sizes $\theta_{1}, \ldots, \theta_{k}$, find an estimate of $\theta$ that is the average of all the $\theta_{i}$ s. As an example, consider the effect size to be the length of manufactured rods, and the $T_{i}$ s to be $k$ measurements of the length of $k$ different rods. In the simplest case, we assume that all rods have exactly the same length $\left(\theta_{1}=\ldots=\theta_{k}=\theta\right)$, and we use a single ruler to measure their lengths so that the variance $\sigma^{2}$ of each measurement is the same. In this case, computing an estimate of $\theta$ and its variance is straightforward with

$$
\begin{equation*}
\hat{\theta}=\bar{T}=\frac{\sum_{i=1}^{k} T_{i}}{k} \tag{B1}
\end{equation*}
$$

and

$$
\begin{equation*}
v=\frac{\sigma^{2}}{k} \tag{B2}
\end{equation*}
$$

Now consider the case when the $k$ measurements are performed with different rulers that have different standard errors $\sigma_{1}$, $\ldots, \sigma_{k}$. We can no longer directly average the $T_{i} \mathrm{~s}$ to obtain an estimate of $\theta$ because they are not equally valuable. In this case, we weight each $T_{i}$ with $w_{i}=1 / \sigma_{i}^{2}$, so low variance measurements have higher weights (Cooper et al., 2009). We then compute the estimate of $\theta$ and its variance as

$$
\begin{equation*}
\hat{\theta}=\bar{T}=\frac{\sum_{i=1}^{k} w_{i} T_{i}}{\sum_{i=1}^{k} w_{i}} \tag{B3}
\end{equation*}
$$

and

$$
\begin{equation*}
v=\frac{1}{\sum_{i=1}^{k} w_{i}} . \tag{B4}
\end{equation*}
$$

This case is often referred to as a fixed-effect model characterized by

$$
\begin{equation*}
T_{i}=\theta+e_{i} \tag{B5}
\end{equation*}
$$

where $\operatorname{Var}\left(e_{i}\right)=\sigma_{i}^{2}$ is the variance of the $i$ th effect size due to estimation error. The fixed-effect model assumes that $e_{i}$ is normally distributed with mean 0 and variance $\sigma_{i}^{2}\left(e_{i} \sim N\left(0, \sigma_{i}^{2}\right)\right.$ ), and the covariance is 0 between $e_{i}$ and $e_{j}$ for all $i \neq j$ (Cov $\left(e_{i}, e_{j}\right)=0$ for $i \neq j$ ).

So far, we have assumed that all rods are identical in length, in which case the fixed-effect model is sufficient. But what if the actual lengths $\theta_{1}, \ldots, \theta_{k}$ are normally distributed around the $\theta$ that we want to estimate? In this case, we need to first determine if there is enough evidence to invalidate the hypothesis that they are created equal. We compute the following homogeneity test statistic for this purpose (Cooper et al., 2009):

$$
\begin{equation*}
Q=\sum_{i=1}^{k}\left[\frac{\left(T_{i}-\bar{T} .\right)^{2}}{\sigma_{i}^{2}}\right]=\sum_{i=1}^{k} w_{i}\left(T_{i}-\bar{T} .\right)^{2} . \tag{B6}
\end{equation*}
$$

If $Q$ exceeds the upper-tail critical value of chi-square at $k-1$ degrees of freedom, the observed variance in effect sizes is significantly greater than what we would expect by chance if all studies shared a common population effect size, and therefore, we reject the null hypothesis (Cooper et al., 2009). Otherwise, there is not enough evidence to reject the null hypothesis, and the fixed-effect model is the correct choice. In the case of rejection, we need to use a random-effect model, characterized by

$$
\begin{equation*}
T_{i}=\theta+u_{i}+e_{i} \tag{B7}
\end{equation*}
$$

where $\operatorname{Var}\left(u_{i}\right)=\sigma_{\theta}^{2}$ is the variance of the effect size due to heterogeneity, and $\operatorname{Var}\left(e_{i}\right)=\sigma_{i}^{2}$ is the variance of the $i$ th effect size due to estimation error. The random-effect model assumes that $e_{i} \sim N\left(0, \sigma_{i}^{2}\right), u_{i} \sim N\left(0, \sigma_{\theta}^{2}\right)$, and $\operatorname{Cov}\left(e_{i}, e_{j}\right)=0$ for $i \neq j$, $\operatorname{Cov}$ $\left(e_{i}, u_{j}\right)=0$ for all $i$ and $j$. In the case of our example, the existence of $u_{i}$ could be due to the fact that these rods are not created equal. We need to take $\sigma_{\theta}^{2}$ into account while computing an estimate of $\theta$; therefore, we need to estimate $\sigma_{\theta}^{2}$. There are several estimators for this purpose, including the Hedges estimator

$$
\begin{equation*}
\hat{\sigma}_{\theta, H}^{2}=\frac{1}{k-1} \sum_{i=1}^{k}\left(T_{i}-\bar{T}\right)^{2}-\frac{1}{k} \sum_{i=1}^{k} \sigma_{i}^{2} \tag{B8}
\end{equation*}
$$

and the DerSimonian-Laird estimator

$$
\begin{equation*}
\hat{\sigma}_{\theta, D S L}^{2}=\frac{Q-(k-1)}{\sum_{i=1}^{k} w_{i}-\frac{\sum_{i=1}^{k} w_{i}^{2}}{\sum_{i=1}^{k} w_{i}}} \tag{B9}
\end{equation*}
$$

where $Q$ is as defined in Equation (B6). The specific conditions under which each estimator should be used are based on Cooper et al. (2009)

$$
\hat{\sigma}_{\theta}^{2}= \begin{cases}\max \left(0, \hat{\sigma}_{\theta, \mathrm{DSL}}^{2}\right) & \text { if homogeneity test is not rejected }  \tag{B10}\\ \hat{\sigma}_{\theta, \mathrm{DSL}}^{2} & \text { if (homogeneity test is rejected, } \left.\hat{\sigma}_{\theta, H}^{2}<0, \hat{\sigma}_{\theta, \mathrm{DSL}}^{2}>0\right) \\ & \text { or (homogeneity test is rejected, } \left.\hat{\sigma}_{\theta, H}^{2}>0, \hat{\sigma}_{\theta, \mathrm{DSL}}^{2}>0, Q \leq 3(k-1)\right) \\ \hat{\sigma}_{\theta, H}^{2} & \text { otherwise. }\end{cases}
$$

After the estimation of $\sigma_{\theta}^{2}$, we compute weights using $w_{i}=1 /\left(\sigma_{\theta}^{2}+\sigma_{i}^{2}\right)$ and use the new $w_{i}$ s in Equations (B3) and (B4).
So far, we have discussed the fixed-effect model and the random-effect model using the example of measuring rods, but how does this relate to the analysis in the paper? In the paper, we assumed that there exists some value corresponding to $\theta$ that describes how exam scores would behave on average with respect to time in asynchronous exams. As we do not know $\theta$, we need to estimate it from the data. We assumed that each asynchronous exam has some slope $\theta_{k}$ that is normally distributed around $\theta$ but cannot be observed directly. Thus, we need to obtain estimates of the $\theta_{k} \mathrm{~s}$, which we do by first having a group of students take the exam asynchronously and then performing ordinary least squares (OLS) regression of scores with respect to time. From the OLS regression, we obtain estimates of the $\theta_{k} \mathrm{~s}$ as $T_{k} \mathrm{~s}$ with standard error $\sigma_{k}$. The OLS regression can be seen as a ruler because we used it to obtain $T_{k}$ with its corresponding error $\sigma_{k}$. These data are what we need to estimate $\theta$.

