

# Parallel finite element computation of contact-impact problems with large deformations

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## Abstract

We present a framework for parallel explicit dynamics simulation of frictional contact-impact problems involving shells and solids on distributed memory architectures. Contact detection is performed in parallel with orthogonal range queries based on a sparse bucket data structure. For contact enforcement a new algorithm is introduced, which exactly satisfies the kinematic impenetrability constraints and conserves linear and angular momenta. The versatility and efficiency of the proposed framework is demonstrated with selected thin-shell and solid applications.

*Keywords:* Parallel finite elements; Contact mechanics; Shells

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## 1. Introduction

The size and complexity of problems, which can be tackled in a parallel computing environment, require particularly robust and efficient contact algorithms. A common approach for deriving contact enforcement methods, such as the penalty or the Lagrangian multiplier method, is to start on the continuum level and to discretize the non-smooth differential equations. This approach leads to the well-known robustness problems, including the non-uniqueness of the normals and non-smoothness of the discretized contact surfaces. In this work, by contrast, we start with the discretized problem and consider the collisions between the triangles of the finite element surface mesh. This approach may be directly derived from the conceptual framework of non-smooth Lagrangian mechanics as discussed by Fetecau et al. [4]. We will demonstrate that the resulting contact enforcement algorithms strictly conserve algorithmic linear and angular momenta.

In large scale computations, the search for possible collisions takes up a significant part of the computing time and can easily become a bottleneck. Contact search is an inherently global problem, as during the computation any surface element can, in principle, collide with any other surface element. In contrast, in explicit codes the computations involving the finite elements are local and the

elements need to communicate only with their immediate neighbors. This basic difference between the contact and the element level computations requires two different partitions as proposed by Attaway et al. [1].

## 2. Contact enforcement

Non-smooth variational mechanics provides an elegant tool for considering the collisions between triangles of the contact surface and for deriving momentum conserving contact enforcement algorithms. The key step in the derivation is the definition of a proper contact indicator function, such as the intersection volume or gap function. In our implementation, the indicator function is defined as the volume enclosed by the surface triangle and the penetrating vertex. In a first step, the collisions are removed by projecting back the penetrating vertex to the surface by using a ray-triangle intersection algorithm. Subsequently, the velocities of the vertices participating at the collision are modified so that momenta and kinetic energy are exactly conserved. The modified velocities are computed by using momentum decompositions with respect to the gradient of the impenetrability constraint function. Similarly, momentum decompositions are used for computing the sliding velocities, which are relevant for computing the frictional contact response. The frictional tractions obey the Coulomb's law and are applied opposite to the sliding velocities. It bears emphasis that the developed algorithm also

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applies to the edge–edge collision case, which is crucial for computing, e.g., fragmentation processes.

### 3. Parallel contact detection

Our parallelization strategy relies on storing the contact surface mesh on all computational nodes in addition to the volume mesh piece assigned by domain partitioning. For the purpose of contact detection, the surface elements are assigned to different computational nodes using the recursive coordinate bisection (RCB) algorithm [3]. The RCB algorithm repeatedly subdivides the domain by choosing an axis-aligned cutting plane so that after the termination of the algorithm each domain has approximately the same number of elements. On each computational node a serial closest point, or orthogonal range query, algorithm is employed for finding possible collisions. The use of the RCB algorithm effectively minimizes the size of the search domains, which has a major influence on the performance of the search algorithm. In problems involving large relative displacements, it is necessary to keep the search domains small by dynamically repartitioning the contact surfaces.

The partitioning of the contact surface leads to small contact search problems on each domain and relatively basic algorithms can be employed. Our orthogonal range query algorithm uses a sparse bucket data structure coupled with a binary search in the sparse direction. In comparison to the classical bucketing algorithms, it leads to significantly reduced storage usage by a slight increase of the access times [5].

### 4. Examples

Our first example concerns the impact of a moving spherical thin-shell ( $R = 0.12$  and  $t = 0.0035$ ) with a thin-plate ( $R = 0.35$  and  $t = 0.0035$ ) at rest. The Neo-Hookean material has a Young modulus of  $E = 210,000$  and a Poisson ratio of  $\nu = 0.3$ . In Fig. 1, the mesh partitioning for 15 computational nodes and three snapshots during the dynamic simulation are shown. The spherical shell as well as plate are discretized with subdivision shell elements [2] and have approx. 40,000 degrees of freedom.

The second example demonstrates the application of the proposed framework to solid dynamic computations.

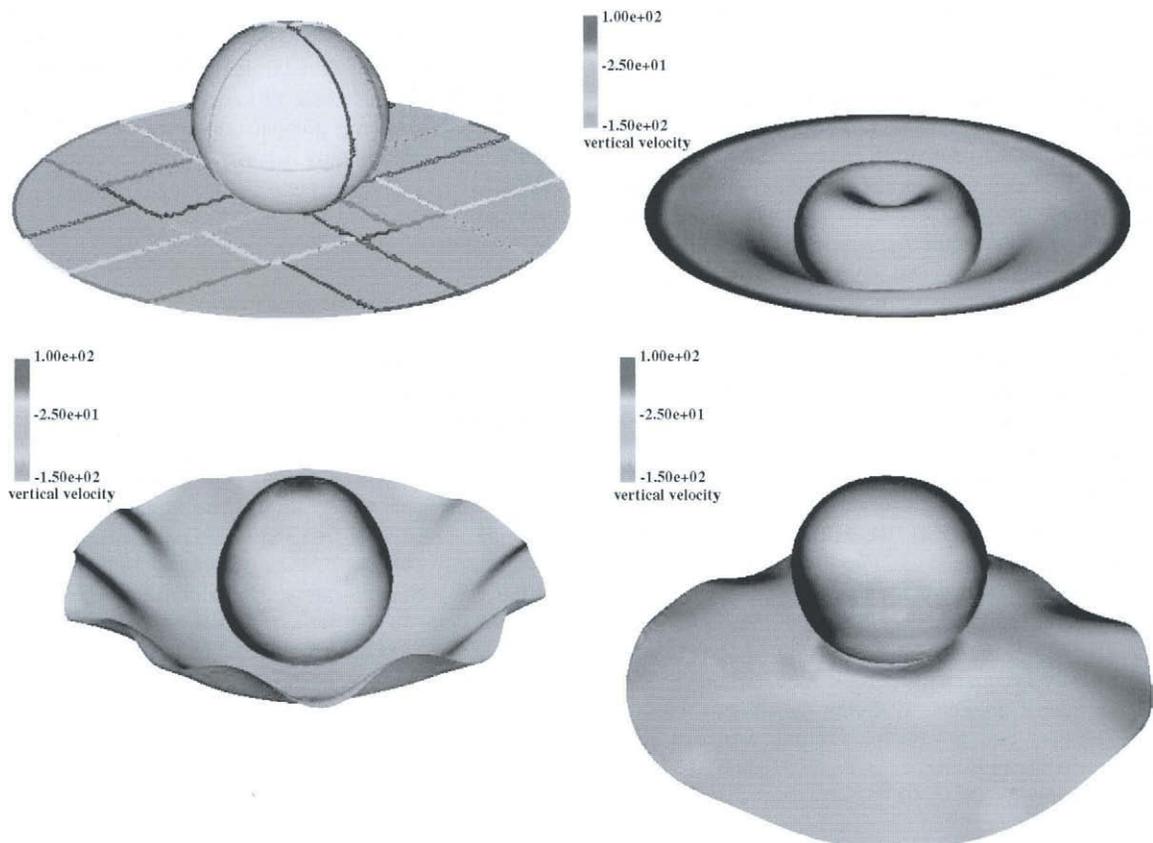


Fig. 1. Impact of a spherical thin-shell on a thin-plate: contact partitioning (top left); deformed configurations.



Fig. 2. Impact of two solid cylinders: contact partitioning (top left); solid partitioning (top right); deformed configurations.

The problem concerns two impacting solid cylinders ( $R = 9.0 \times 10^{-4}$  and  $L = 9.0 \times 10^{-3}$ ) with an impact speed of 200. The Neo-Hookean material has a Young modulus of  $E = 210 \times 10^9$  and a Poisson's ratio of  $\nu = 0.3$ . The impact speed of both cylinders is 200. In Fig. 2 top row, the contact

and solid partitions and on bottom row the deflected shapes are shown.

The parallel scalability of the presented examples is being investigated on our 100-node Beowulf cluster and on terascale ASCI platforms.

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